

Visible and hidden sectors in a model with Maxwell and Chern-Simons gauge dynamics

Edwin Ireson^a, Fidel A. Schaposnik^b and Gianni Tallarita^c

^a *Department of Physics, Swansea University
Singleton Park, Swansea, SA2 8PP, UK*

^b *Departamento de Física, Universidad Nacional de La Plata/IFLP/CICBA
CC 67, 1900 La Plata, Argentina*

^c *Departamento de Ciencias, Facultad de Artes Liberales
Universidad Adolfo Ibáñez Santiago 7941169, Chile.*

July 7, 2016

Abstract

We study a $U(1) \times U(1)$ gauge theory discussing its vortex solutions and supersymmetric extension. In our set-up one of two Abelian gauge fields is governed by a Maxwell term, the other by a Chern-Simons term. The two sectors interact via a BF gauge field mixing and a Higgs portal term that connects the two complex scalars. We also consider the supersymmetric version of this system which allows to find for the bosonic sector BPS equations in which an additional real scalar field enters into play. We study numerically the field equations finding vortex solutions with both magnetic flux and electric charge.

1 Introduction

Gauge fields coupled to charged scalars in models with two weakly connected sectors have recently attracted much attention both in high energy and condensed matter physics. Concerning the former, many extensions of the Standard Model (SM) propose the existence of additional product gauge groups in models where there are particles transforming under the new gauge fields and not under the usual local symmetries (See [1]-[2] and references therein). Also, different supersymmetry extensions of the SM include a hidden sector intended to break supersymmetry (SUSY) leading to acceptable superpartner masses. This is the case of the supersymmetric model proposed in [3] with two $U(1)$ gauge symmetries in which the gauge field in the hidden sectors communicates SUSY breaking to the visible sector through a gauge kinetic term.

Two possible gauge-invariant and renormalizable mixings between the fields of two Abelian Higgs model sectors can be considered, either coupling the gauge fields or the complex scalars. In the former case working in $3 + 1$ space-time dimensions one has the so called gauge kinetic mixing with an interaction Lagrangian between the two sectors originally introduced in [4]-[6]

$$L_{mix} = \xi F_{\mu\nu}[A] F^{\mu\nu}[B] \quad (1)$$

where $F_{\mu\nu}$ is the field strength associated to each one of the two $U(1)$ gauge fields A_μ, B_μ .

In the case of $2 + 1$ space-time, relevant for describing systems in planar physics, there is a second gauge mixing which takes the form of a BF term,

$$L_{mix} = \xi' \epsilon^{\mu\nu\alpha} B_\mu F_{\nu\alpha}[A] \quad (2)$$

Concerning the scalar-scalar mixing, one has the so-called Higgs portal interaction L_{HP} , originally introduced in refs. in [7]-[8],

$$L_{HP} = \zeta |\phi^2| |\eta|^2 \quad (3)$$

with ϕ and η the complex scalars. It is interesting to note that if one takes ϕ as the scalar doublet of the Standard Model and η as a Goldstone boson, a fractional increase in the number of relativistic species (dark radiation; e.g., neutrinos) can be produced [9].

Classical string-like solutions to the field equations of models with two Abelian Higgs sectors connected by a gauge mixing interaction have been found and their possible relevance in the context of dark matter has been discussed [10]-[12].

Supersymmetric extensions of gauge theories coupled to scalars having two sectors have been also considered in [13] in connection with lower-energy precision experiments. Now, supersymmetry necessarily require both types of mixing with couplings appropriately related, precisely in the way required for the existence of self-dual first order (BPS) equations in the bosonic sector [14]-[16]. In the case of models with visible and hidden sectors this has been studied for a $3 + 1$ dimensional $U(1) \times U(1)$ model with gauge dynamics governed by a Maxwell term in each sector in ref.[18] and for a $2 + 1$ dimensional model with Chern-Simons gauge dynamics [19].

Regarding condensed matter applications, $2 + 1$ dimensional Abelian Higgs models with two sectors, in one of which the $U(1)$ symmetry remains unbroken, interesting effects in connection with superconductivity were discussed [12],[20].

It is the purpose of the present work to consider the case in which the gauge dynamics is a $2 + 1$ dimensional model in which gauge dynamics in each sector corresponds to different kinetic terms, namely a Maxwell kinetic term in one sector and a Chern-Simons term in the other. The model could be of interest both in the analysis of a $3 + 1$ field theory at high temperature and also in the study of parity violation phenomena in planar physics.

The plan of the paper is the following. We first consider (in section 2) a bosonic $U(1) \times U(1)$ gauge theory with spontaneous symmetry breaking in both sectors which are connected through BF gauge mixing and Higgs portal interactions. We then proceed in section 3 to a SUSY extension of the model, finding first order self-dual equations associated with the bosonic sector. In section 4 we analyze the numerical solutions of both models, finally giving in section 5 a summary of the results and conclusions.

2 The $U(1) \times U(1)$ model

We shall consider a $2 + 1$ dimensional Abelian gauge theory with dynamics governed by the following $U(1)_M \times U(1)_{CS}$ Lagrangian

$$L_{U(1) \times U(1)} = L_{MH} + L_{CSH} + L_{mix} \quad (4)$$

Here L_{MH} and L_{CSH} are Maxwell-Higgs (MH) and Chern-Simons-Higgs (CSH) Lagrangians,

$$L_{MH} = -\frac{1}{4}F^{\mu\nu}[A]F_{\mu\nu}[A] + \frac{1}{2}|D_\mu[A]\phi|^2 - V_M[\phi] \quad (5)$$

$$L_{CSH} = \frac{\kappa}{4}\epsilon^{\alpha\beta\gamma}B_\alpha F_{\beta\gamma}[B] + \frac{1}{2}|D_\mu[B]\eta|^2 - V_{CS}[\eta] \quad (6)$$

with (A_μ, ϕ) and (B_μ, η) the gauge and complex scalar fields in each sector. Field strengths and covariant derivatives for the MH sector are given by

$$F_{\mu\nu}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu\phi[A] = (\partial_\mu + ieA_\mu)\phi \quad (7)$$

and an analogous formulæ for the B, η fields in the CSH sector, with the coupling constant e replaced by g . Concerning the scalar potentials, we shall consider for the MH sector the usual quartic symmetry breaking potential,

$$V_M = \frac{\lambda_M}{4}(|\phi|^2 - \phi_0^2)^2 \quad (8)$$

which for $\lambda_M = e^2/8$ (usually dubbed the Bogomolny point) exhibits, in the case of the ordinary Abelian Higgs model, first order self-dual equations [17]-[14].

Concerning the potential in the hidden sector, since the model is defined in $(2+1)$ space-time dimensions one has the choice (on renormalizability grounds) to have a fourth-order or sixth-order potential. We shall choose the latter option, since only in that case do first-order self-dual equations exist for the case of a Chern-Simons-Higgs theory with just one sector [21]-[22]. We then have

$$V_{CS} = \frac{\lambda_{CS}}{4}|\eta|^2(|\eta|^2 - \eta_0^2)^2 \quad (9)$$

In the case of an ordinary Chern-Simons-Higgs model the Bogomolny point corresponds to $\lambda_{CS} = 4g^4/\kappa^2$

The two sectors interact through the mixing Lagrangian L_{mix} , consisting of a BF-like coupling between gauge fields, and a Higgs portal coupling the two complex scalars:

$$L_{mix} = \xi\epsilon^{\mu\alpha\beta}A_\mu\partial_\alpha B_\beta + \zeta(|\phi|^2 - \phi_0^2)(|\eta|^2 - \eta_0^2) \quad (10)$$

We have chosen the Higgs portal in the way it arises in supersymmetric models with the same gauge dynamics in each sector (either MH-MH or CSH-CSH) [18]-[19]. As we shall see in the next section such kind of interaction also holds in the present case but of course supersymmetry forces the coupling constant ζ to take a particular value in terms of coupling constants e, g, ξ and κ .

In our conventions, gauge and scalar fields as well as gauge couplings e, g have mass dimensions 1/2 while the rest of the coupling constants have mass dimensions 1.

The field equations associated to Lagrangian (4) read in the Lorenz gauge

$$\square A_\mu = i\frac{e}{2}(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) - e^2 |\phi|^2 A_\mu - \xi \varepsilon_{\mu\alpha\beta} \partial^\alpha B^\beta \quad (11)$$

$$\kappa \varepsilon_{\mu\alpha\beta} \partial^\alpha B^\beta = i\frac{g}{2}(\eta^* \partial_\mu \eta - \eta \partial_\mu \eta^*) - g^2 |\eta|^2 B_\mu - \xi \varepsilon_{\mu\alpha\beta} \partial^\alpha A^\beta \quad (12)$$

$$\square \phi + 2ieA_\mu \partial^\mu \phi - e^2 A_\mu A^\mu \phi = -\frac{\lambda_M}{2}(|\phi|^2 - \phi_0^2)\phi + \zeta(|\eta|^2 - \eta_0^2)\phi \quad (13)$$

$$\square \eta + 2igB_\mu \partial^\mu \eta - g^2 B_\mu B^\mu \eta = -\frac{\lambda_{CS}}{4}(|\eta|^4 - \eta_0^4)\eta + \zeta(|\phi|^2 - \phi_0^2)\eta \quad (14)$$

Let us start by analyzing the Gauss laws resulting from eqs.(11)-(12),

$$\square A_0 = j_0[\phi, A; e] - \xi \varepsilon_{ij} \partial^i B^j \quad (15)$$

$$\kappa \varepsilon_{ij} \partial^i B^j = j_0[\eta, B; g] - \xi \varepsilon_{ij} \partial^i A^j \quad (16)$$

Here $i, j = 1, 2$ and the scalar currents are defined as

$$j_\mu[\phi, A; e] = i\frac{e}{2}(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) - e^2 |\phi|^2 A_\mu \quad (17)$$

and an analogous formula for $j_\mu[\eta, B; g]$.

Integrating over space eqs.(15)-(16) we find

$$Q_A = \xi \Phi_B \quad (18)$$

$$Q_B = \xi \Phi_A + \kappa \Phi_B \quad (19)$$

where we have defined electric charges and magnetic fluxes according to

$$Q_A = \int d^2x j_0[\phi, A; e], \quad Q_B = \int d^2x j_0[\eta, B; g], \quad (20)$$

$$\Phi_A = \int d^2x \varepsilon_{ij} \partial^i A^j, \quad \Phi_B = \int d^2x \varepsilon_{ij} \partial^i B^j \quad (21)$$

It is interesting to note that finite energy electrically charged vortex configurations can only exist in the Abelian Higgs model when the gauge dynamics is governed by a Chern-Simons term. In the present case, vortices associated to the Maxwell term get a non trivial charge through the BF term that mixes both gauge fields without provoking any energy divergence, as we shall see below.

Since we are interested in vortex solutions, we shall make a static axially symmetric ansatz. As already signalled, in the case of the ordinary Maxwell-Higgs system one has to choose $A_0 = 0$ since otherwise no finite energy solutions exist. In contrast, for the Chern-Simons-Higgs model vortices should necessarily carry electric charge since otherwise the magnetic flux vanishes. In the present case eqs. (18)-(19) force to include both A_0 and B_0 .

Indeed, for static configurations j_0 as defined in eq.(21) leads to an electric charge of the form

$$Q_A = -e^2 \int d^2x |\phi^2| A_0 \quad (22)$$

so that taking $A_0 = 0$ would lead, in view of eq.(18) to $\Phi_B = 0$. We then propose the following Ansatz

$$\begin{aligned} \phi &= f(r) \exp(-in\varphi) , & A_\varphi &= -\frac{1}{r}A(r) , & A_r &= 0 , & A_0 &= A_0(r) \\ \eta &= q(r) \exp(-ik\varphi) , & B_\varphi &= -\frac{1}{r}B(r) , & B_r &= 0 , & B_0 &= B_0(r) \end{aligned} \quad (23)$$

With this Ansatz the equations for the spatial components of the gauge fields read

$$A'' - \frac{A'}{r} + 2r\xi B'_0 - e(n + eA)f^2 = 0 \quad (24)$$

$$\kappa B'_0 + 2\xi A'_0 - \frac{g}{r}(k + gB)q^2 = 0 \quad (25)$$

Concerning the Gauss laws, they take the form

$$A''_0 + \frac{A'_0}{r} + \frac{2\xi}{r}B' - e^2 A_0 f^2 = 0 \quad (26)$$

$$B_0 - \frac{2\xi A' + \kappa B'}{rg^2q^2} = 0 \quad (27)$$

Finally, for the radial scalar field equations we have

$$\begin{aligned} f'' + \frac{f'}{r} - f \left(\frac{(n + eA)^2}{r^2} - e^2 A_0^2 + \lambda_M (f^2 - \phi_0^2) + 2\zeta (\eta_0^2 - q^2) \right) &= 0, \\ q'' + \frac{q'}{r} - q \left(2\frac{(k + gB)^2}{r^2} - 2g^2 B_0^2 + 4\zeta (\phi_0^2 - f^2) + \lambda_{CS} ((\eta_0^2 - 2q^2)^2 - q^4) \right) &= 0. \end{aligned} \quad (28)$$

The appropriate boundary conditions for finite energy vortex solutions are

$$\begin{aligned} A(0) &= 0 , & A_0(0) &= \alpha_0 , & \phi(0) &= 0 , & B(0) &= 0 , & B_0(0) &= \beta_0 , & \eta(0) &= 0 \\ A(\infty) &= -n/e , & A_0(\infty) &= 0 , & \phi(\infty) &= \phi_0 , & B(\infty) &= -k/g , & B_0(\infty) &= 0 , & \eta(\infty) &= \eta_0 \end{aligned}$$

where α_0 and β_0 are constants.

Concerning magnetic fluxes and electric charges we have from ansatz (23)

$$\Phi_A = (2\pi/e)n , \quad Q_A = \xi(2\pi/g)k \quad (29)$$

$$\Phi_B = (2\pi/g)k , \quad Q_B = \kappa(2\pi/g)k + \xi(2\pi/e)n \quad (30)$$

We shall present the numerical solutions to the second order radial field equations (24)-(28) in section 5. It should be stressed that no first order BPS equations can be found for the case of Lagrangian $L_{U(1) \times U(1)}$ (4). Indeed, working à la Bogomolny it is not possible to accommodate the energy as a sum of squares plus a topological term. We shall then proceed in the next section to the $\mathcal{N} = 2$ SUSY extension of a model with the same gauge dynamics searching for the existence of self-dual equations. As we shall see, this will imply the presence in the bosonic sector of an additional scalar field with non-trivial dynamics, a scalar potential different from the one we introduced above and a reduction of the number of independent coupling constants. With all this, we shall see that Bogomolny first order equations can be found.

3 The $\mathcal{N} = 2$ supersymmetric $U(1) \times U(1)$ Model

We shall consider two $\mathcal{N} = 2$ supersymmetric sectors in $2+1$ dimensions, one with Maxwell-Higgs dynamics, the other one with Chern-Simons-Higgs dynamics with gauge symmetry breaking in both sectors. These two sectors will be coupled using the CS-like superfield mixing introduced in [19].

Concerning the Maxwell-Higgs sector, we shall introduce the vector superfield V_M , which in the the Wess-Zumino gauge is composed of a gauge field A_μ , a 2-component complex spinor ψ_h , a real scalar field M and an auxiliary scalar field D , and for the matter content, one introduces a scalar superfield $\Phi = (\phi, \lambda, F)$. containing a complex scalar ϕ , a complex fermion λ and an auxiliary field F (which we will omit as there is no superpotential).

The $\mathcal{N} = 2$ supersymmetric Maxwell-Higgs action written in components reads (see [18] and references therein)

$$S_{MH} = \int d^3x \left(-\frac{1}{4} F_{\mu\nu}[A] F^{\mu\nu}[A] + |D_\mu[A]\phi|^2 + \frac{1}{2} D^2 + e(|\phi|^2 - \phi_0^2) D \right. \\ \left. + \frac{1}{2} \partial_\mu M \partial^\mu M - e^2 M^2 |\phi|^2 + \frac{i}{2} \bar{\psi} \not{D}[A] \psi + \frac{i}{2} \bar{\lambda} \not{D} \lambda + e M \bar{\psi} \psi - i\sqrt{2}e (\bar{\psi} \lambda \phi - \phi^* \bar{\lambda} \psi) \right) \quad (31)$$

We have included a Fayet-Iliopoulos term (last one in the first line) to implement gauge symmetry breaking. The γ -matrices are taken as the Pauli matrices with $\gamma^0 = \sigma^3$, $\gamma^i = i\sigma^i$.

The Action S_{MH} is invariant under the following transformations with infinitesimal anticommuting complex parameters η ,

$$\delta\phi = \sqrt{2}\bar{\eta}\psi, \quad \delta\psi = -\sqrt{2}\eta(i\gamma_\mu D^\mu[A] - eM)\phi, \\ \delta M = i(\bar{\eta}\lambda - \bar{\lambda}\eta), \quad \delta A_\mu = i(\bar{\eta}\gamma_\mu\lambda - \bar{\lambda}\gamma_\mu\eta), \\ \delta D = \partial^\mu(\eta\gamma_\mu\bar{\lambda} - \bar{\eta}\gamma_\mu\lambda), \quad \delta\lambda = \eta(i\epsilon^{\mu\nu\rho}\partial_\mu A_\nu\gamma_\rho + \not{D}M - iD) \quad (32)$$

In the case of the supersymmetric Chern-Simons-Higgs sector we also introduce a vector superfield V_{CS} , composed (in the Wess-Zumino gauge) of a gauge field B_μ , a 2-component complex spinor χ , a real scalar field N and the auxiliary scalar field d .

Concerning the matter content, one introduces a scalar superfield $\Psi = (\eta, \sigma, f)$ containing a complex scalar η , a complex fermion σ and an auxiliary field f (again ignored).

The $\mathcal{N} = 2$ supersymmetric action written in terms of component fields (see [19] and references there) takes the form

$$S_{CSH} = \int d^3x \left(\kappa \left(\frac{1}{4} \epsilon^{\mu\nu\rho} B_\mu \partial_\nu B_\rho + \bar{\chi}\chi + dN \right) + |D_\mu[B]\eta|^2 + i\bar{\sigma}\gamma^\mu \not{D}_\mu[B]\sigma - g^2 N^2 \eta^2 \right. \\ \left. + gd(|\eta|^2 - \eta_0^2) + gN\bar{\sigma}\sigma - i\sqrt{2}e(\bar{\sigma}\chi\phi + \phi^*\bar{\chi}\sigma) \right) \quad (33)$$

Concerning SUSY variations, one has

$$\delta B_\mu = i(\bar{\epsilon}\gamma_\mu\chi - \bar{\chi}\gamma_\mu\epsilon), \quad \delta N = i(\bar{\epsilon}\chi - \bar{\chi}\epsilon) \quad (34)$$

$$\delta\eta = \sqrt{2}\bar{\epsilon}\sigma, \quad \delta d = \partial_\mu(\epsilon\gamma^\mu\bar{\chi} - \bar{\epsilon}\gamma^\mu\chi) \quad (35)$$

$$\delta\chi = \epsilon(i\epsilon^{\mu\nu\rho}\gamma_\rho\partial_\mu B_\nu - id + \gamma^\mu\partial_\mu N), \quad \delta\sigma = \sqrt{2}\epsilon(-i\gamma^\mu D_\mu[B]\eta + gN\eta) \quad (36)$$

Finally, we choose, as the $\mathcal{N} = 2$ supersymmetric mixing between the two sectors, the one introduced in [19] which reads: in components

$$S_{mix} = \xi \int d^3x \left(\frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}[A] + (\bar{\lambda}\chi + \bar{\chi}\lambda) + (DN + dM) \right) \quad (37)$$

The first term in (37) is a BF-like kinetic gauge mixing while the last two terms, once the auxiliary fields are eliminated using their equations of motion will give rise to a Higgs portal interaction.

The complete action governing the dynamics of our $\mathcal{N} = 2$ Maxwell-Chern-Simons-Higgs model is then defined as

$$S_{\mathcal{N}=2} = S_{CSH} + S_{MH} + S_{int} \quad (38)$$

with S_{CSH} , S_{MH} , S_{int} given by eqs.(33), (31) and (37) respectively

To determine the explicit form of the symmetry breaking potential in action (38) we consider the equations associated to the auxiliary fields D, d and to N

$$\begin{aligned} \delta D : D &= -e(|\phi|^2 - \phi_0^2) - \xi N \\ \delta d : \kappa N &= -g(|\eta|^2 - \eta_0^2) - \xi M \\ \delta N : \kappa d &= 2g^2\eta^2 N - \xi D \end{aligned} \quad (39)$$

Solving these algebraic equations we express the values of fields D and N in terms of the remaining scalars, ϕ , η and M . Note that in the ordinary supersymmetric extension of the Maxwell-Higgs theory the M -field can be put to zero while in the present case it has non-trivial dynamics (see eq.(31)) Inserting the values of D, d, N obtained in eq.(39) in action (38) we get, for the symmetry breaking potential $V_{eff}(\phi, \eta, M)$

$$\begin{aligned} V_{eff}(\phi, \eta, M) &= \frac{g^2}{\kappa^2} (g(|\eta|^2 - \eta_0^2) + \xi M)^2 |\eta|^2 \\ &+ \frac{1}{2} \left(e(|\phi|^2 - \phi_0^2) - \frac{\xi}{\kappa} (g(|\eta|^2 - \eta_0^2) + \xi M) \right)^2 + e^2 M^2 |\phi|^2 \end{aligned} \quad (40)$$

Compared with the potential chosen in the model of the previous section, composed of the fourth order symmetry breaking potential (8) for the Maxwell-Higgs sector, the sixth order one (9) for the Chern-Simons-Higgs sector and a Higgs portal term in (10), the resulting potential V_{eff} exhibits two important differences. Primo, the coupling constants for each one of the two terms in the potential are not independent parameters but, forced by SUSY, they are fixed in terms of the gauge coupling constants e, g and the gauge mixing coupling constant ξ . Secundo, an additional real field M enters in the bosonic sector Lagrangian with couplings to the other scalars.

In the case of the ordinary supersymmetric Maxwell-Higgs model the equations of motion of the real scalar M has the solution $M = 0$ which is then chosen so as to end with the Lagrangian of the ordinary Abelian Higgs model in the bosonic sector. In contrast, the addition of a Chern-Simons term promotes its role to that of a field with non-trivial dynamics, as originally observed in [26]-[27]. The same phenomenon takes place in the $U(1) \times U(1)$ model, in which Maxwell and Chern-Simons term belong to different sector which are connected through a BF gauge mixing term and a Higgs portal.

The potential V_{eff} has a minimum value $V_{eff} = 0$ for which both $U(1)$ symmetries are broken

$$V_{eff} [|\phi| = \phi_0, |\eta| = \eta_0, M = 0] = 0 \quad (41)$$

Degenerate with this there is also a second symmetric minimum for $\xi \neq 0$

$$V_{eff} [\eta = 0, \phi = 0, M = (g/\xi)\phi_0^2] = 0 \quad (42)$$

provided the following relation between ϕ_0 and η_0 holds,

$$e\phi_0^2 = -2g\frac{\xi}{\kappa}\eta_0^2 \quad (43)$$

Concerning the Lagrangian for the bosonic sector, it takes the form

$$\begin{aligned} L_{bos} = & -\frac{1}{4}F_{\mu\nu}[A]F^{\mu\nu}[A] + |D_\mu[A]\phi|^2 + \frac{1}{2}\partial_\mu M\partial^\mu M - e^2 M^2 |\phi|^2 + \frac{\kappa}{4}\epsilon^{\mu\nu\rho}B_\mu\partial_\nu B_\rho \\ & + |D_\mu[B]\eta|^2 + \frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho}[A] - V_{eff}[\phi, \eta, M] \end{aligned} \quad (44)$$

In order to obtain the BPS equations associated to this bosonic Lagrangian we can proceed as usual by equating to zero the SUSY variation for fermion fields [15]-[16]. We get

$$\epsilon^{ij}\partial_i A_j = \pm (-e(|\phi|^2 - \phi_0^2) + \xi g(|\eta|^2 - \eta_0^2) + \xi M) , \quad D_\pm[A]\phi = 0$$

$$\epsilon^{ij}\partial_i B_j = \pm \frac{1}{\kappa} \left(\xi e(|\phi|^2 - \phi_0^2) - \frac{1}{\kappa} (2g^2\eta^2 + \xi^2) \right) (g^2(|\eta|^2 - \eta_0^2) + \xi M) , \quad D_\pm[B]\eta = 0$$

$$A_0 = \pm M , \quad B_0 = \mp \frac{1}{\kappa} (g(|\eta|^2 - \eta_0^2) + \xi M) \quad (45)$$

with $D_\pm = D_1 \pm iD_2$. These equations, together with the Gauss law (17) saturate the Bogomolny bound for the energy. The upper (lower) sign corresponds to positive (negative) value of the magnetic charges.

4 Vortex solutions

We first discuss the numerical solutions to the radial field equations (11)-(14) for the model with dynamics governed by Lagrangian (4) introduced in Section 2. The numerical solver involves a second order central finite difference procedure with accuracy $\mathcal{O}(10^{-4})$. Field profiles of the obtained solutions for different ranges of parameters, are shown in figures 1 to 3.

Figure 1 shows the field profiles' dependence on the gauge mixing parameter ξ . One can see in figures 1(a) and 1(b) that the Higgs field profiles are almost insensitive to changes in ξ , growing from 0 to its vacuum expectation values in the same way as in the ordinary MH and CSH models. In contrast the B_A magnetic field behavior drastically departs with

respect to type-II superconductivity vortices. Indeed, one can see in figure 1(c) that as ξ grows the shape of the profile changes and already for $\xi = 0.2$ (measured in units of ϕ_0) it develops a second maximum which corresponds to a ring surrounding the origin. This latter maximum is in fact larger than that in the core.

Concerning the electric field E_A , it is of course absent for $\xi = 0$ but, as ξ grows, it becomes non-trivial with the shape of two rings with opposite electric field signs. Also in the CS-Higgs sector the magnetic field behavior radically changes: for $\xi = 0$ B_B vanishes at the origin as in the ordinary CSH model. Now, for $\xi \neq 0$ it has a negative value near the origin and then it becomes positive as r grows. In contrast the electric field E_B has a different behavior as shown in figure 1(f): it vanishes at the origin independently of the ξ value and solely the position and height of the maximum changes as ξ varies. The behavior of the electric fields can be better understood by analyzing the bosonic sector of a supersymmetric version of the model, see next section.

Figure 2 shows the dependence of the field on the value of the Higgs portal coupling constant ζ with the other parameters fixed. Again the Higgs field profiles do not exhibit a significant difference as ζ varies (figures 2(a) and 2(b)). Concerning the magnetic and electric fields their behavior is qualitatively similar to the previously discussed case, in which the ξ value was changed.

In order to determine how the CS term affects the field behaviors we have analyzed their dependence on the Chern-Simons factor κ keeping all other parameters fixed. As can be seen in figure 3 the behavior of the scalar field profiles shows a greater dependence on κ compared to that found when changing the mixing parameters. As for the magnetic and electric fields, the behavior is rather similar to those resulting from changing ξ or ζ , this showing that the presence of the CS term is one of the determinant factors in the physical properties of the model we are discussing.

We have also studied the dependence of the vortex solutions on coupling constants $e, g, \lambda_M, \lambda_{CS}$ without finding any notable change when their values were varied over a wide range.

In the case of the $\mathcal{N} = 2$ SUSY model the search for axially symmetric vortex solutions for its bosonic sector should be done for boundary conditions leading to the symmetry breaking vacuum (see eq. (41)) together with the condition $M(0) = A(0)$ and $M(r) \rightarrow 0$ as $r \rightarrow \infty$. We have solved the Bogomolny equations (45) associated to Lagrangian (44) using the same numerical approach as the one in the precedent section finding that the profiles for the scalar, electric and magnetic fields do not differ significantly from the ones obtained for the model discussed in section 3. We then solely present in figure 4 the results for the real scalar field M whose presence was forced by supersymmetry.

5 Summary and discussion

In this work we have studied the case of $U(1) \times U(1)$ gauge theories with spontaneously broken symmetry in which the dynamics of one of the $U(1)$ gauge fields is dictated by a Maxwell term, the other one by a Chern-Simons term. One can then consider that there two sectors, a visible and a hidden one, as those in models discussed in the context of dark matter, supersymmetry breaking and other relevant problems in particle physics. Since the

models are defined in $2+1$ space-time dimensions they can be taken as toy models for realistic models or the high temperature limit of a realistic quantum field theory. They could also be relevant in connection with condensed matter physics for the study of superconductivity, quantum Hall effect and topological insulators.

In section 2 we considered a purely bosonic theory in which the interaction between the two sectors include a gauge field kinetic mixing and a Higgs portal. An interesting point to notice has to do with the existence of electrically charged vortex solutions. As it is well known, the Abelian Higgs model with a Maxwell term does not support such solutions which in contrast should be there if the Chern-Simons action governs the gauge field dynamics. In the present case, because of the gauge mixing term, vortices in the Maxwell sector are necessarily charged while the charge in the CS sector gets a contribution from the flux in the other sector (eqs.(18)-(19)) this leading to the relations (29)-(30) between quantized magnetic fluxes and electric charges in the two sectors.

The fact that we could not find self-dual equations associated to the $U(1) \times U(1)$ Lagrangian (4) was to be expected. Already for a $U(1)$ model including both Maxwell and CS terms it was shown in [26] that an additional neutral scalar field should be added in order to be able to write the vortex energy as a sum of squares plus a topological term which then gives the Bogomolny bound and the corresponding BPS equations.

Another way to find BPS equations is to proceed to the $\mathcal{N} = 2$ supersymmetric extension of a purely bosonic Lagrangian this implying the existence of the additional scalar field which in the case of the Maxwell-Chern-Simons dynamics does not have the trivial solution $M = 0$ [27]. The same happens in the case of our $U(1) \times U(1)$ model as was shown in section 3, where we found that M couples not only with the complex scalar of its sector but also with the hidden one leading to the effective potential (40)). Following the usual procedure we have obtained from the supersymmetric fermion field variations the BPS equations that, together with the Gauss law, allow to find the minima of the energy for each topological sector.

We present in section 4 a detailed discussion of the numerical solutions that we have obtained for the two models considered in this work. One of the main features stemming from the gauge mixing of the two sectors is the onset of a purely radial electric field for the vortex configurations corresponding to the Maxwell sector because of the presence of a CS action in the other sector. In fact, the numerical results show that the presence of the CS term is one of the determinant factors controlling the behavior of the system. As it is well-known, because of the parity anomaly in $2 + 1$ dimensions, Chern-Simmons terms can be induced by radiative quantum effects, even if they are not present as bare terms in the original Lagrangian. This is a key property in $2 + 1$ bosonization [28]-[29] with many possible applications to condensed matter problems like those related to quantum Hall effect and topological insulators. In this context the effective action resulting from integration of fermions coupled to one of the gauge fields in $U(1) \times U(1)$ models could correspond to one of those studied in this work hence leading to topological configurations of the type described here.

Acknowledgments: G.T. is funded by Fondecyt grant No. 3140122 . F.A.S. is associated to CICBA and financially supported by PIP-CONICET, PICT-ANPCyT, UNLP and CICBA grants.

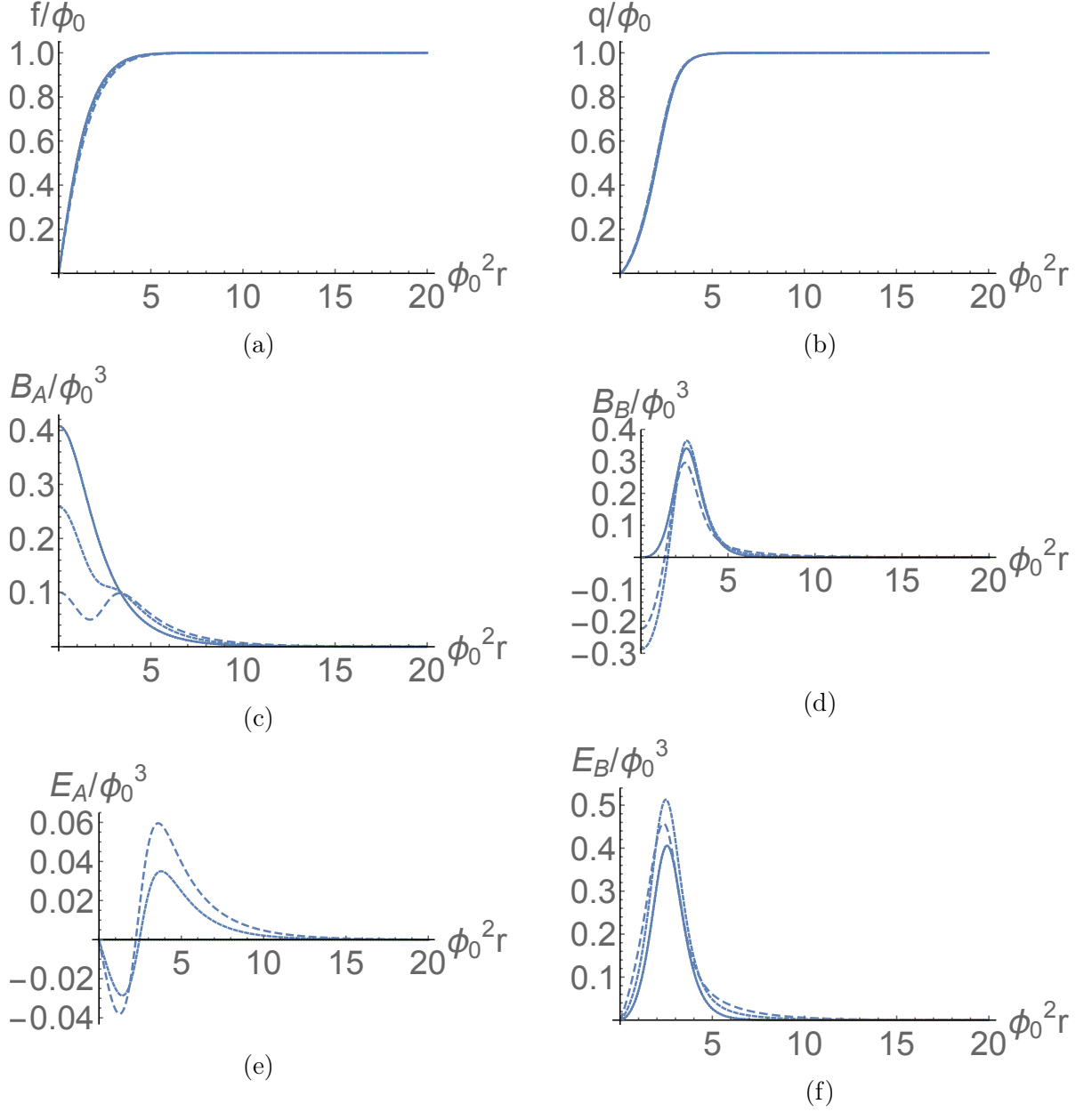


Figure 1: Field profiles for different values of the gauge mixing parameter, $\xi = 0., 0.1, 0.2$. Progressively larger dashing corresponds to larger values of ξ . Full line corresponds to the case of no mixing. All dimensional parameters are measured in units of ϕ_0 . We have chosen $n = k = 1$ and $\eta_0 = \lambda_M = \lambda_{CS} = 1$; $e = g = 0.5$, $\zeta = 0.1$, $\kappa = 0.18$.

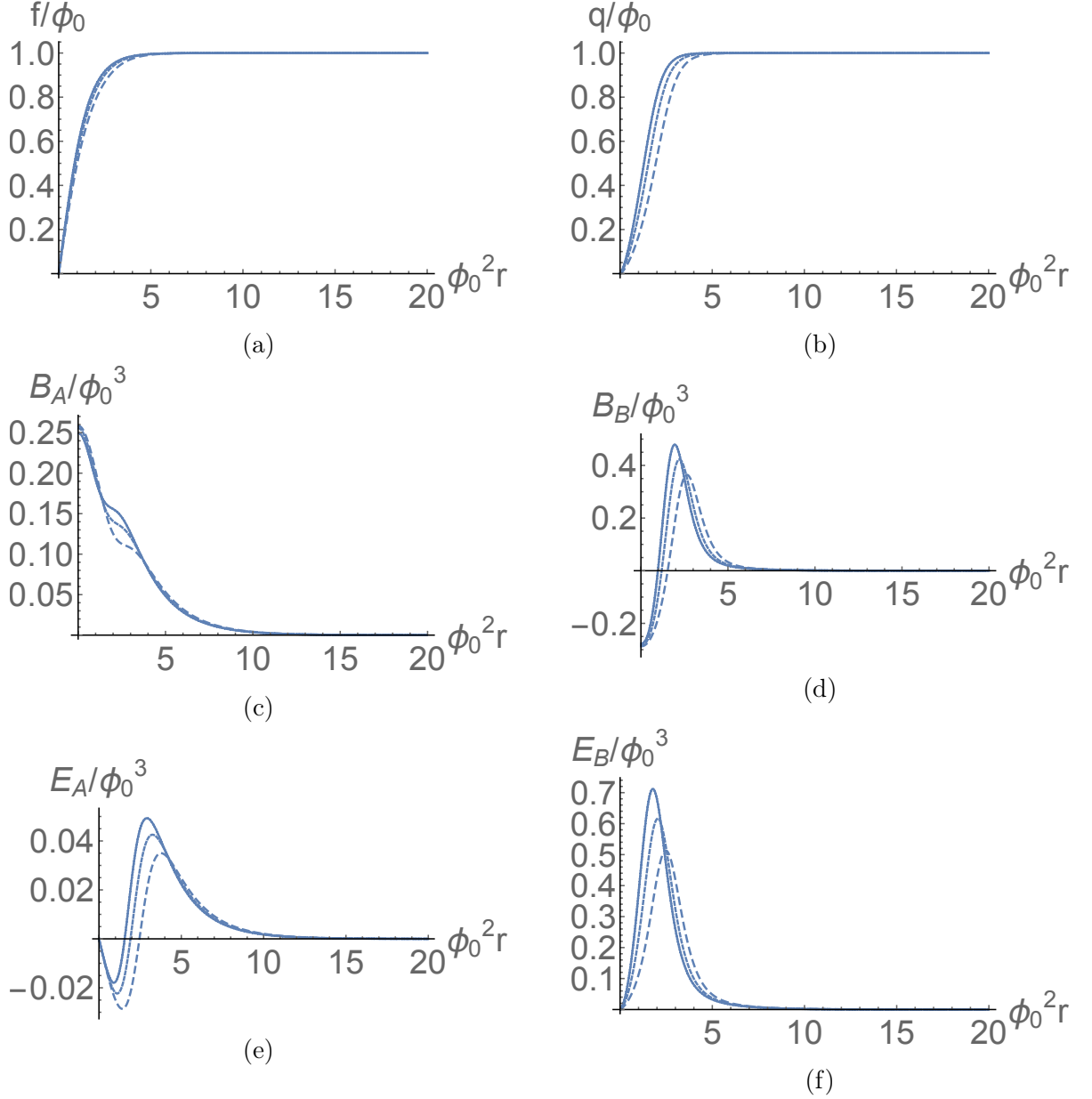


Figure 2: Field profiles for different values of the Higgs portal mixing, $\zeta = 0., 0.05, 0.1$. Progressively larger dashing corresponds to larger values of ζ . Full line corresponds to the case of no mixing. All dimensional parameters are measured in units of ϕ_0 . We have chosen $n = k = 1$ and $\eta_0 = \lambda_M = \lambda_{CS} = 1$, $e = g = 0.5$, $\xi = 0.1$, $\kappa = 0.18$.

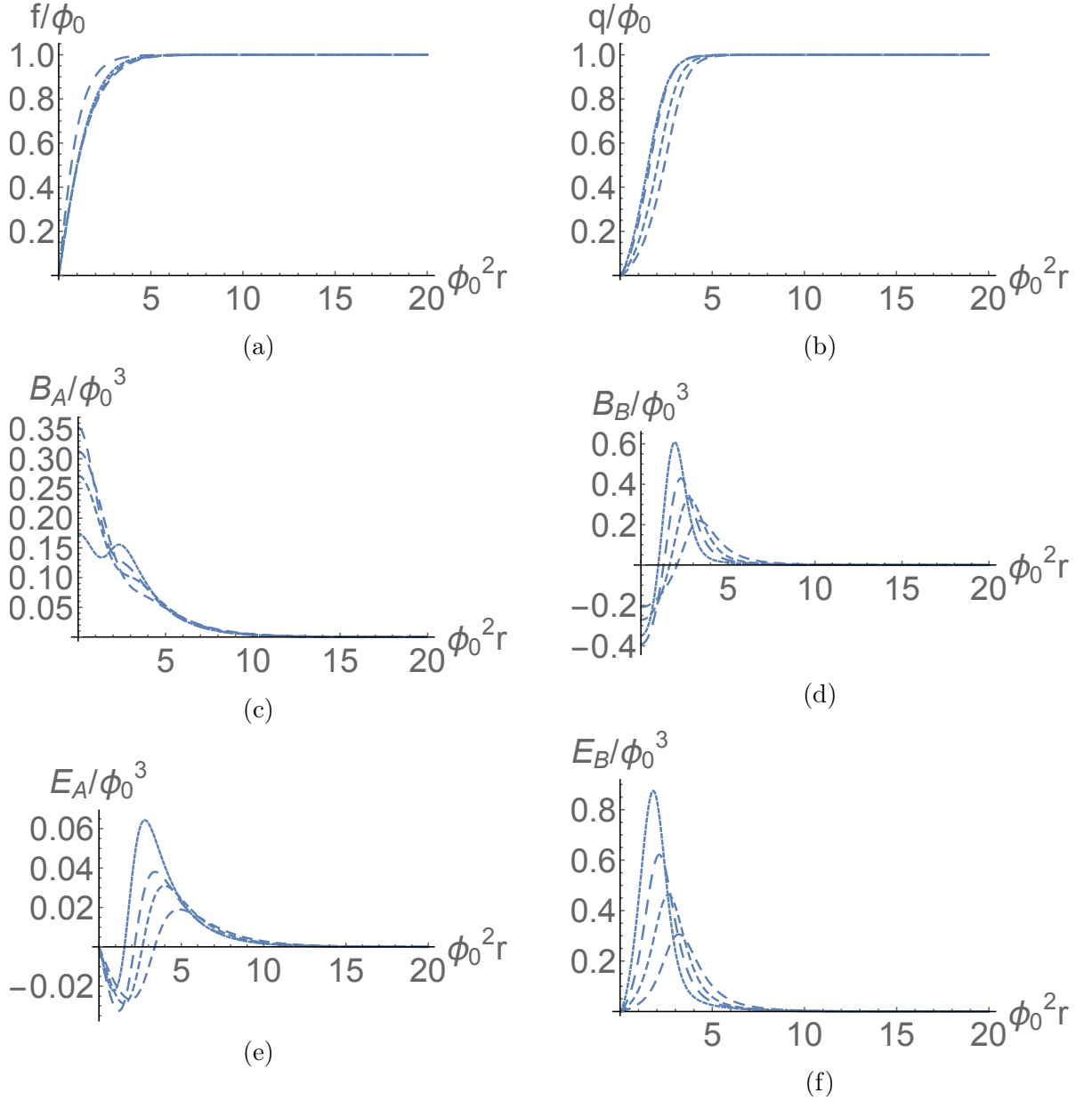


Figure 3: Field profiles for different values of the Chern-Simons coefficient, $\kappa = 0.1, 0.2, 0.3, 0.4$. Progressively larger dashing corresponds to higher κ . All dimensional parameters are measured in units of ϕ_0 . We have chosen $n = k = 1$, $\eta_0 = \lambda_M = \lambda_{CS} = 1$ and $e = g = 0.5$, $\xi = \zeta = 0.1$

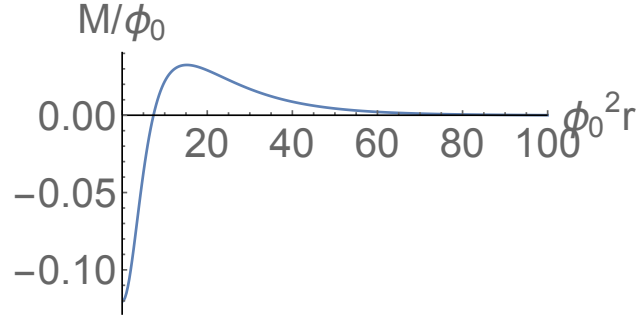


Figure 4: Solution profile for the M field. The solution corresponds to for $n = k = 1$, $\eta_0 = \lambda_M = \lambda_{CS} = 1$ and $e = g = 0.1$, $\kappa = 0.05$, $\xi = 0.1$ (measured in units of ϕ_0).

References

- [1] J. Jaeckel and A. Ringwald, *Ann. Rev. Nucl. Part. Sci.* **60** (2010) 405 [arXiv:1002.0329 [hep-ph]].
- [2] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo and A. Ringwald, *JCAP* **1206** (2012) 013 [arXiv:1201.5902 [hep-ph]].
- [3] K. R. Dienes, C. F. Kolda and J. March-Russell, *Nucl. Phys. B* **492** (1997) 104 doi:10.1016/S0550-3213(97)80028-4, 10.1016/S0550-3213(97)00173-9 [hep-ph/9610479].
- [4] L. B. Okun, *Sov. Phys. JETP* **56** (1982) 502 [*Zh. Eksp. Teor. Fiz.* **83** (1982) 892].
- [5] P. Galison and A. Manohar, *Phys. Lett. B* **136** (1984) 279.
- [6] B. Holdom, *Phys. Lett. B* **166** (1986) 196.
- [7] V. Silveira and A. Zee, *Phys. Lett. B* **161** (1985) 136.
- [8] B. Patt and F. Wilczek, hep-ph/0605188.
- [9] S. Weinberg, *Phys. Rev. Lett.* **110** (2013) no.24, 241301
- [10] B. Hartmann and F. Arbabzadah, *JHEP* **0907** (2009) 068
- [11] Y. Brihaye and B. Hartmann, *Phys. Rev. D* **80** (2009) 123502.
- [12] P. Arias, F. A. Schaposnik, *JHEP* **1412** (2014) 011 doi:10.1007/JHEP12(2014)011 [arXiv:1505.06705 [hep-th]].
- [13] D. E. Morrissey and A. P. Spray, *JHEP* **1406** (2014) 083
- [14] H. J. de Vega and F. A. Schaposnik, *Phys. Rev. D* **14** (1976) 1100.
- [15] E. Witten and D. I. Olive, *Phys. Lett. B* **78** (1978) 97.
- [16] J. D. Edelstein, C. Núñez and F. Schaposnik, *Phys. Lett. B* **329** (1994) 39

- [17] E. B. Bogomolny, Sov. J. Nucl. Phys. **24** (1976) 449 [Yad. Fiz. **24** (1976) 861].
- [18] P. Arias, E. Ireson, C. Núñez and F. Schaposnik, JHEP **1502** (2015) 156
- [19] P. Arias, E. Ireson, F. A. Schaposnik and G. Tallarita, Phys. Lett. B **749** (2015) 368
- [20] M. M. Anber, Y. Burnier, E. Sabancilar and M. Shaposhnikov, Phys. Rev. D **93** (2016)
- [21] J. Hong, Y. Kim, and P. Y. Pac, Phys. Rev. Lett. **64** (1990) 2230.
- [22] R. Jackiw and E. J. Weinberg, Phys. Rev. Lett. **64** (1990) 2234.
- [23] P. Fayet, Il Nuovo Cimento A **31** (1976) 626.
- [24] A. Salam and J. Strathdee, Nucl. Phys. B **97** (1975) 293.
- [25] P. Fayet and J. Iliopoulos, Phys. Lett. B **51** (1974) 461.
- [26] C. k. Lee, K. M. Lee and H. Min, Phys. Lett. B **252** (1990) 79.
- [27] B. H. Lee, C. k. Lee and H. Min, Phys. Rev. D **45** (1992) 4588.
- [28] E. H. Fradkin and F. A. Schaposnik, Phys. Lett. B **338** (1994) 253
- [29] E. Fradkin, E. F. Moreno and F. A. Schaposnik, Phys. Lett. B **730** (2014) 284